Indian Statistical Institute, Bangalore

M. Math. First Year

Second Semester - Algebra II

Final Exam | Duration: 3 hours | Date : April 21, 2016

Max Marks: 100

Note: Answer all questions. Your answers should be clear, complete and to the point.

1. (a) With precise definitions of the terms used, state the Galois correspondence for finite Galois extensions. [8]

   (b) Determine the Galois groups of the polynomials $x^4 - 2$ and $x^3 + x + 1$ over Q. [10+8]

2. Give an example of extension field K of a field k for each of the following:

   (a) K is a separable extension of k, but not a normal extension o f k.

   (b) K is a normal extension of k, but not a separable extension.

   (c) Characteristic of k is nonzero and $K \backslash k$ contains both algebraic elements as well as transcendental elements over k.
   Justify your answer. [5+5+5]

3. Give an example of an irreducible polynomial of degree 5 over $\mathbb{F}_2$. [10]

4. Define an algebraically closed field. Prove that any two algebraic closures of a field k are k-isomorphic. [3+7]

5. Let G be the group of all linear automorphisms of an n dimensional vector space V over the finite field of p elements (p is taken to be prime). Let P be a subgroup of G of order a power of p. Show that, for the natural action of G on V, and for every integer $0 \leq d \leq n$, there exists a subspace A of V such that $g(A) = A$ for all $g \in P$. [12]

6. Determine nonabelian groups of order 12 up to isomorphism. [12]

7. Show that A_5 does not contain a subgroup of order 20. Can it have a subgroup of order 15? [10+5]